Optimal Perturbations with the MITgcm:

MOC & tropical SST in an idealized ocean

Laure Zanna (Harvard)

Patrick Heimbach, Eli Tziperman, Andy Moore

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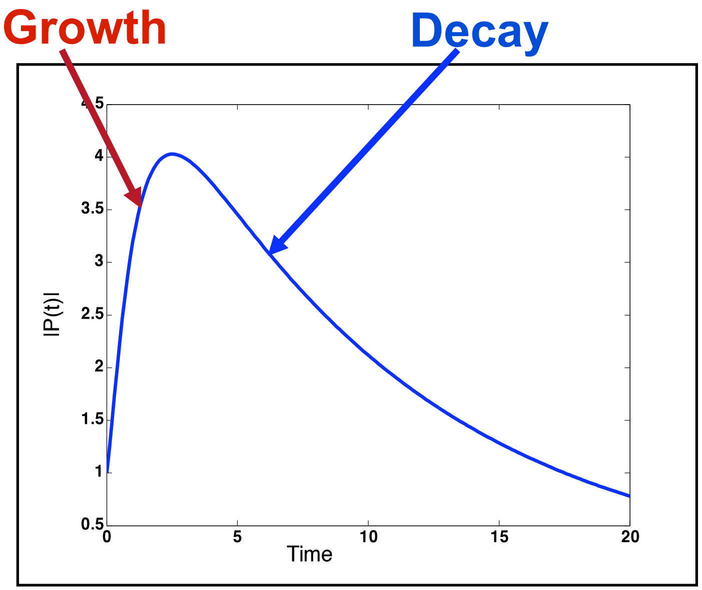
Which perturbations can lead to the most efficient growth?

- In stable systems, perturbations can grow significantly before eventually decaying due to the interaction of several non-orthogonal modes *(e.g., Farrell 1988, Trefethen 1993)*



- Optimal initial conditions = singular vectors ➔ fastest growing perturbations leading to an amplification of a given quantity *(e.g., Buizza & Palmer 1995)*

- Relevance = e.g., climate stability & variability, sensitivity, predictability and error growth, building an observational system *(e.g., Marotzke et al 1999, Moore & Kleeman 1999, Moore et al 2004)*

Objectives

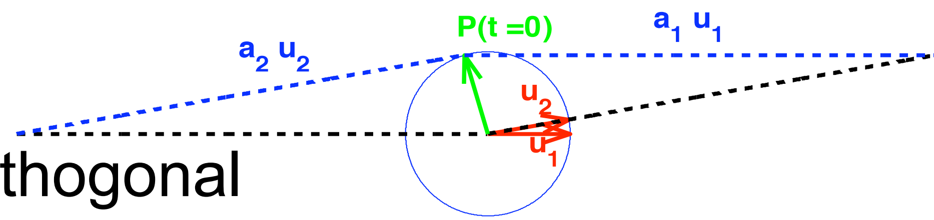
- Spatial structure of the optimal initial conditions leading to the maximum growth of the physical quantities: heat flux, MOC, tropical SST, kinetic & available potential energy, …

- Identification of the growth mechanism for the perturbations & implications for stability and variability of ocean & climate

- → Can observed ocean variability be explained as small amplitude damped linear dynamics excited by atmospheric & other stochastic forcing via non-normal growth?

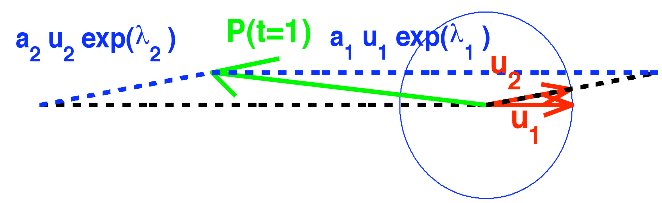
Transient Amplification

Stable linear system $\frac{d\vec{P}(t)}{dt} = A\vec{P}(t), \quad \vec{P}(t) \to 0 \;\; as \quad t \to \infty$



If A is non-normal $AA^T \neq A^T A$
then eigenvectors $\vec{u}_i$ are not orthogonal
➔ may lead to transient amplification

(2D) solution at time $\boldsymbol{\tau}$:

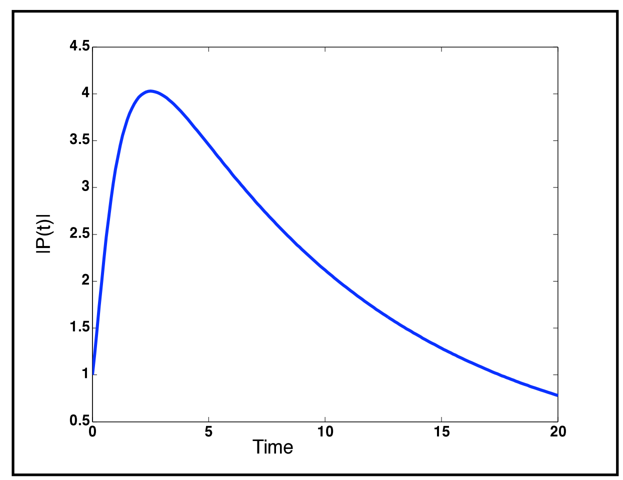


$$\vec{P}(\tau) = a_1\vec{u}_1 e^{\lambda_1 \tau} + a_2\vec{u}_2 e^{\lambda_2 \tau}$$

$$If\; \lambda_2 << \lambda_1 < 0,\; then\;\; a_2\vec{u}_2 e^{\lambda_2} \to 0 \;\; quickly$$

$$leaving\; mostly\; \vec{P}(t=1) \approx a_1\vec{u}_1 e^{\lambda_1}$$

$$eventually \quad \vec{P}(t \to \infty) \to 0$$



Transient amplification: Interaction of non orthogonal eigenmodes b/c of

(1) Partial initial cancellation

(2) Different decay rates

(e.g. Farrell, 1988, Trefethen, 1993)

Evaluating the Optimal Initial Conditions: Eigenvalue Problem

- Full nonlinear model linearized about steady state

$$\frac{d\vec{P}'}{dt} = A\big|_{\overline{P}}\vec{P}', \quad \vec{P}'(t) = e^{At}\vec{P}_0' = B(t)\vec{P}_0'$$

most fluid dynamical systems are non-normal

- Maximize MOC or SST anomalies at time $t=\tau$ to find optimal initial conditions $\vec{P}_0'$

$$\max_{\vec{P}_0'}\left\{\vec{P}_0'^T B^T X B \vec{P}_0' - \lambda\left(\vec{P}_0'^T Y \vec{P}_0' - 1\right)\right\}$$

- Equivalent to a generalized eigenproblem for optimal initial conditions $\vec{P}_0'$

$$\boxed{B^T X B \vec{P}_0' = \lambda Y \vec{P}_0'}$$

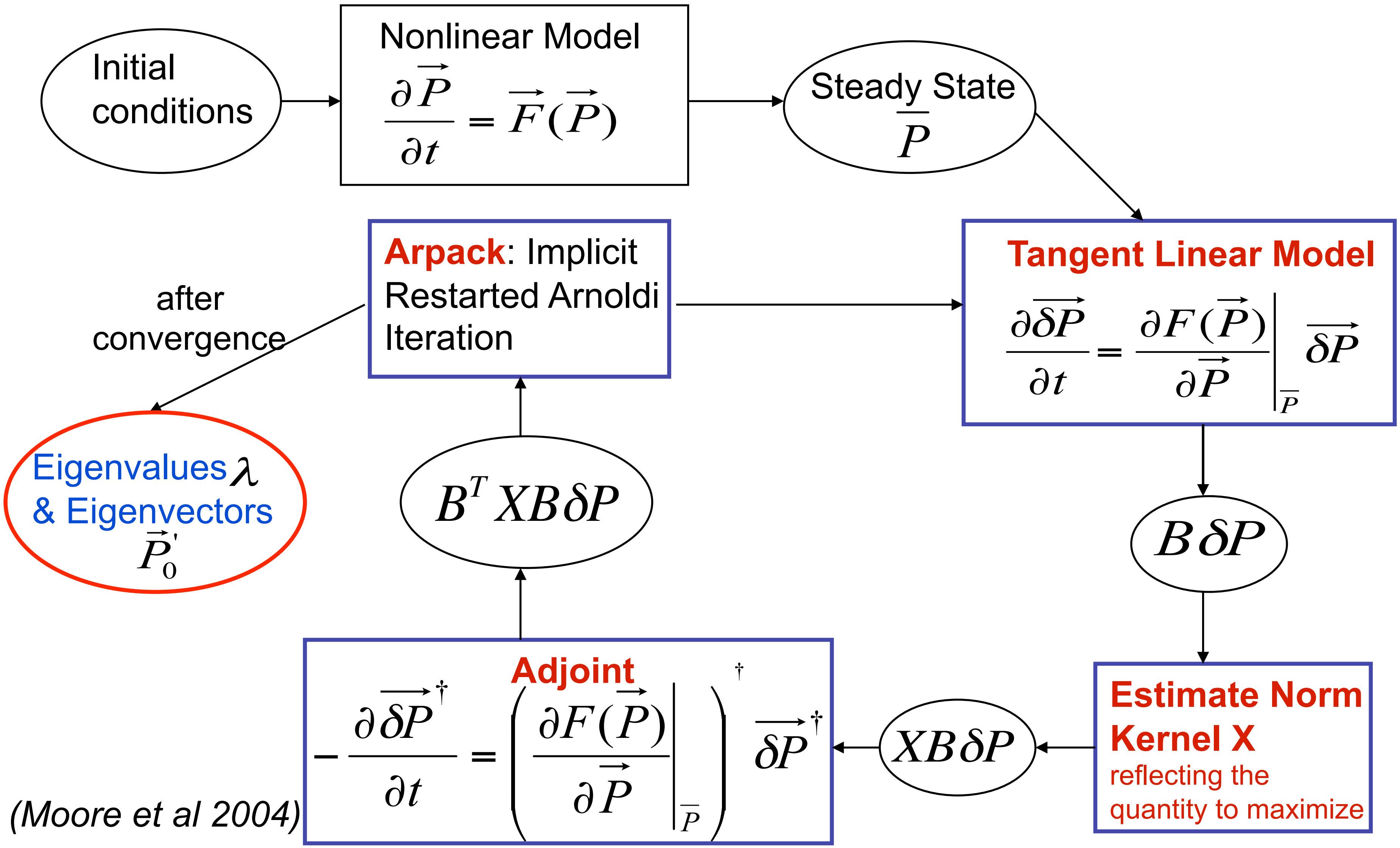
MOC or Tropical SSTs at $t=\tau$

T and S anomalies at $t=0$

(e.g. Farrell, 1988)

Methodology: Optimals using the MITgcm

Finding optimal initial conditions $\longrightarrow$ Solving for eigenvectors $\vec{P}_0'$ & eigenvalues λ of the generalized eigenproblem $(e^{A\tau})^T X e^{A\tau} (= B^T X B)$

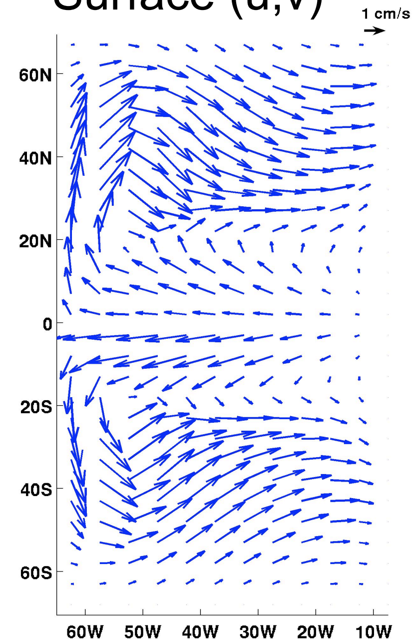


MITgcm: Mean State

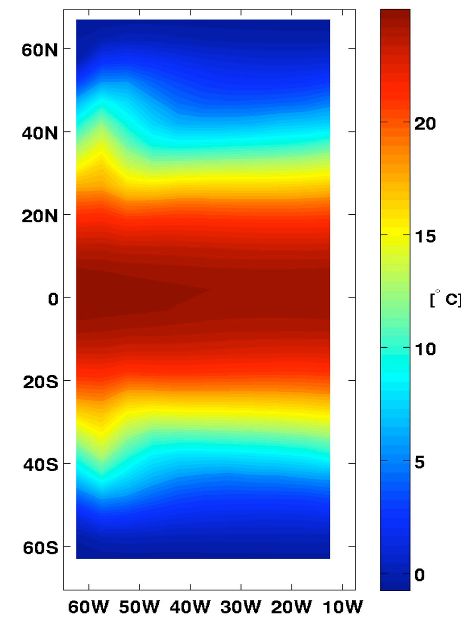
- Primitive, hydrostatic, incompressible, Boussinesq eqns on a sphere
- Configuration: rectangular double-hemisphere ocean basin, coarse resolution 3°x3°, 15 vertical levels, flat topography
- Convection=Implicit diffusion
- Annual mean forcing
- Mixed boundary conditions

(e.g., Marshall et al. 1997; http://mitgcm.org)

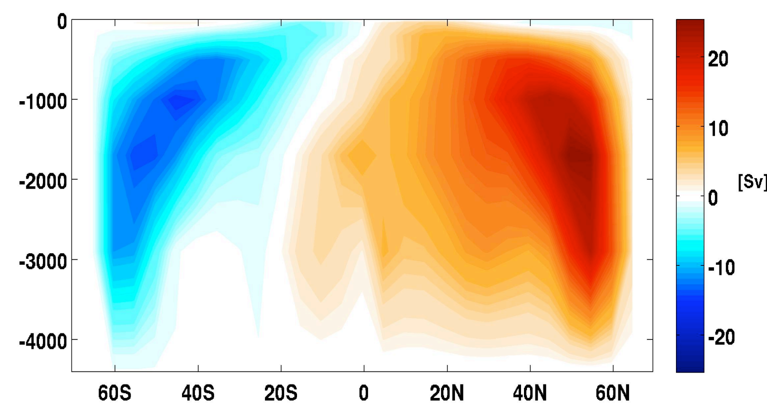
Surface (u,v)



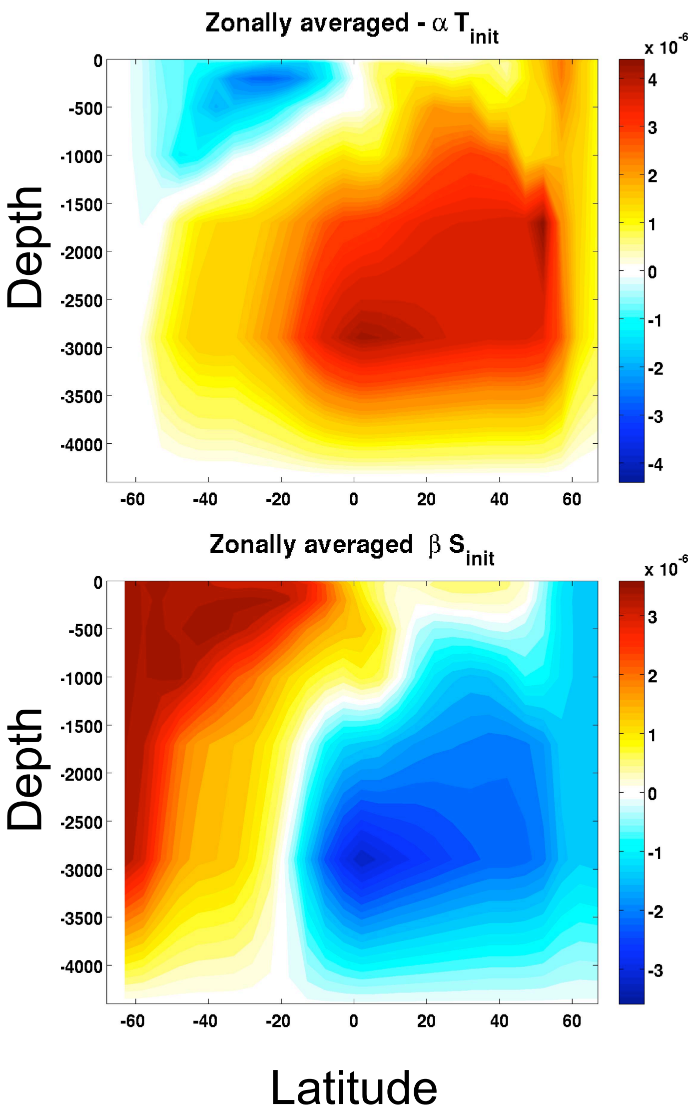
SST



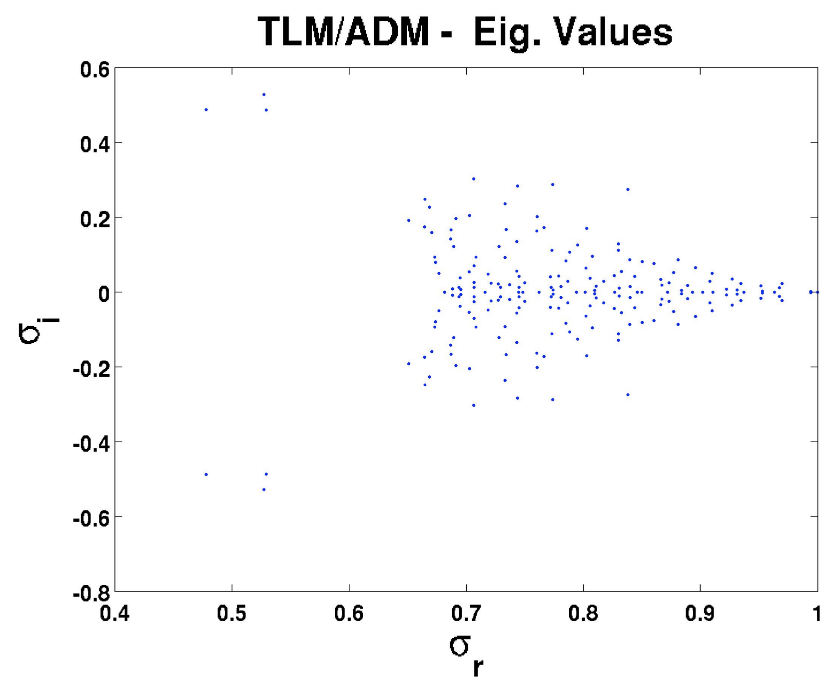
MOC



Stability of the Tangent Linear Model



TLM least damped mode with decay time of 800 yrs



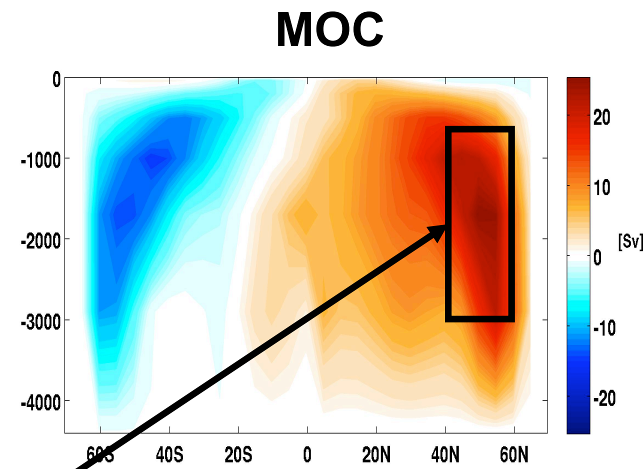
TLM/ADM Imag. vs Real eig. values for t=2 yrs

Transient growth of MOC anomalies:

preliminary results *(Zanna et al, in prep)*

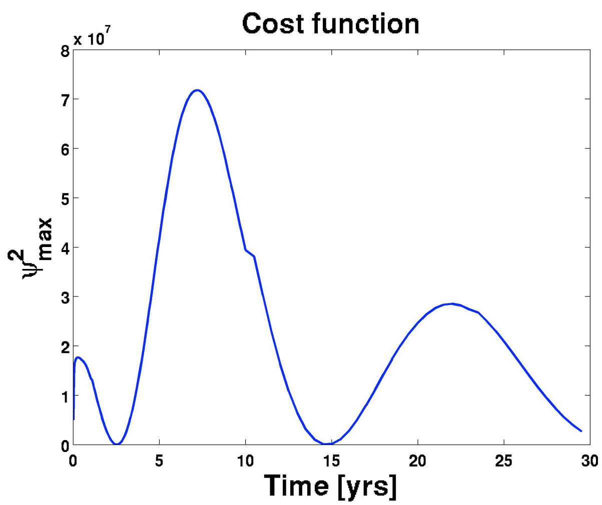
- MOC stability & variability: salinity advective feedback, from interannual to multidecadal, NAO-gyre interaction *(e.g, Marotzke 1990, Marshall et al 2001)*

- Few studies on transient amplification of MOC (e.g., Lohman & Schneider 1999, *Zanna & Tziperman 2005, Sevellec et al 2008)*



- Cost function: sum of the square of the MOC anomalies 50N-60N, 1000m-3500m *(Bugnion & Hill 2006)*

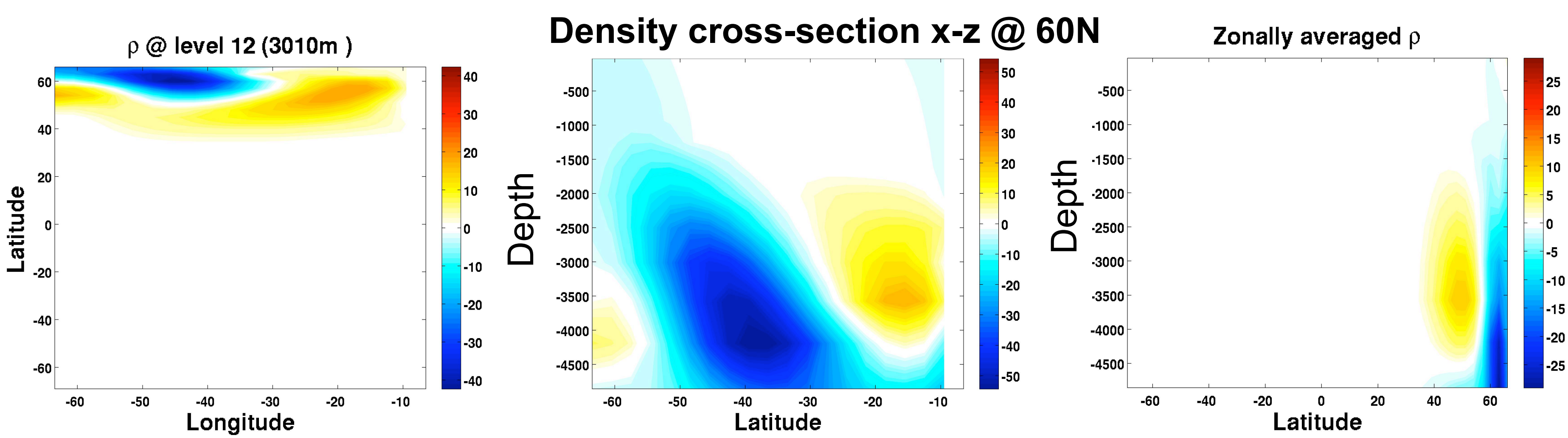
- Results: Optimal i.c of T & S lead to growth of MOC anomalies after ~8 yrs associated with non-orthogonal oscillatory modes. Growth factor ~8.



Cost funtion as fct of time when initializing TLM with optimals

Transient growth of MOC anomalies:
Initial conditions

- Signal mostly in NH with baroclinic structure
- Strong signal in the deep ocean with additive contribution of T & S to buoyancy
- T & S necessary for growth *(unlike Marotzke 1990; Sevellec et al 2008)*
- Similarities with unstable oscillatory mode under fixed flux of *Raa & Dijkstra (2002)*

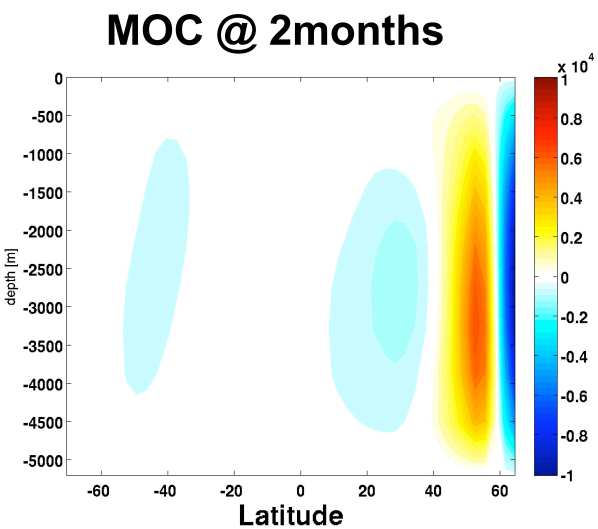


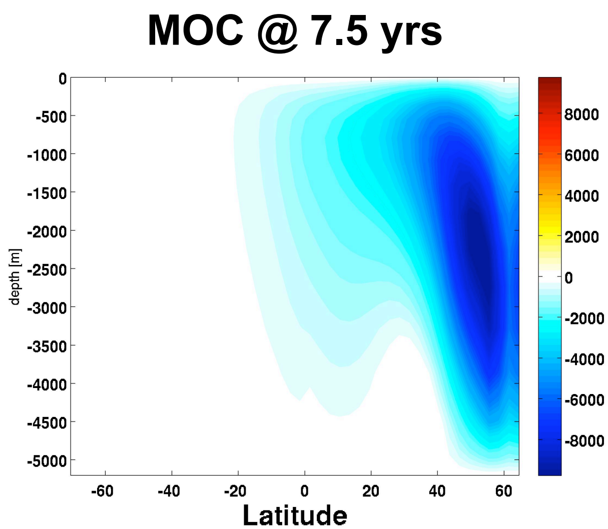
Growth Mechanism:
Preliminary & simplified results

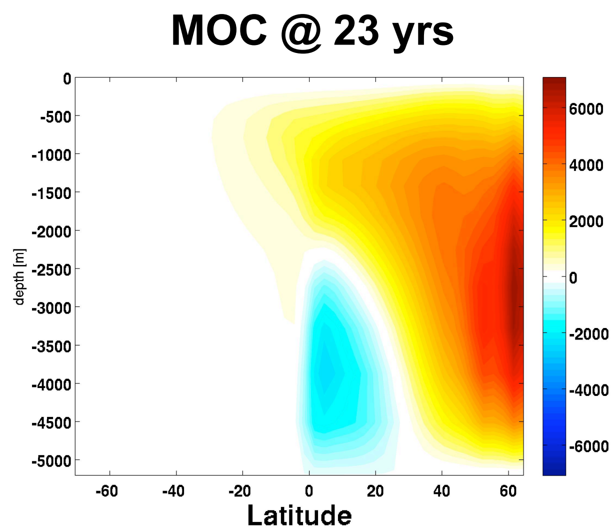
- Several decaying oscillatory modes under mixed boundary conditions but no growth of individual modes

- Growth = change of APE due advection of density perturbations by the mean flow creating N-S & E-W buoyancy gradient

- Oscillation = phase difference btw the N-S & E-W buoyancy gradients

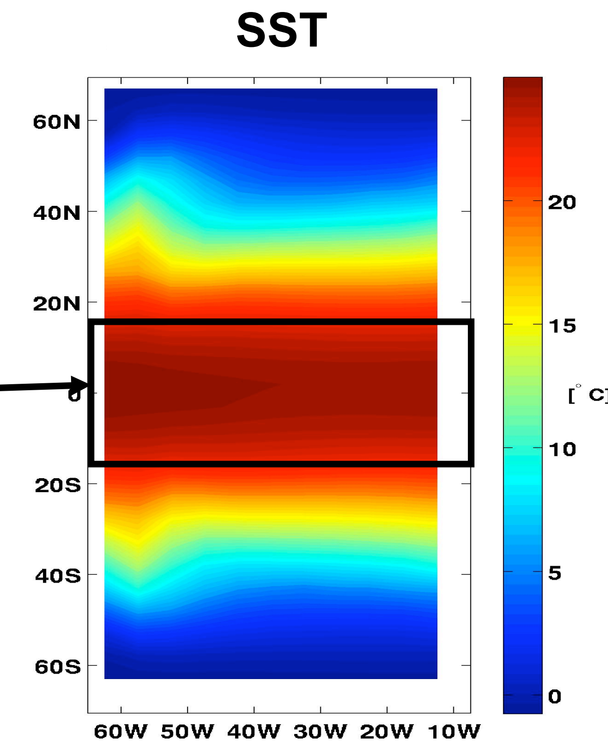


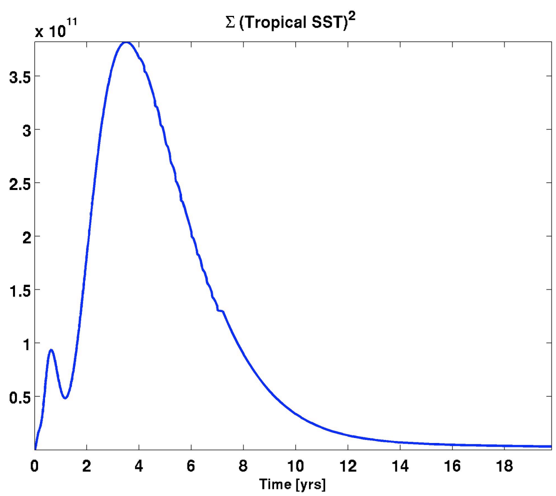




Exciting Tropical SST anomalies

- Tropical Atlantic Variability mechanisms → air-sea interaction or connected to seasonal cycle *(e.g., Chang, Xie & Carton, Jochum et al)*
- Cost function: sum of the square of the SST anomalies btw 15S & 15 N
- Optimal i.c.= deep salinity anomalies near the western boundary @ 30N/S & 50 N/S





Sum of squares of Tropical SST anomalies as fct of time

- Results: Optimal growth of tropical SST anomalies after 3.5-4 yrs. Initial 0.1 ppt → 0.4 C.
- Mechanism: geostrophic adjustment → Coastal & Equatorial Kelvin waves *(Zanna et al, submitted to JPO)*

Conclusions

- *Small* perturbations → *Large* amplification on interannual timescales without unstable modes
- Identification of new mechanisms leading to growth of perturbations
- Preferred anomalies located in the deep ocean:
- Non-normal dynamics can possibly play a dominant role in generating variability on interannual timescales if excited by stochastic forcing

…

Conclusions

- From Box models to GCMs:
  - non-normality of the propagator mainly due to advection & surface boundary conditions
  - Faster time scales in 3D (<10yrs) than in 2D models (decades)
  - More complex dynamics in full GCM & possibility to explore different physical quantities: MOC, energy, heat flux, etc
- Idealized MITgcm:
  - Eigenvectors of the TLM &
  - Singular vectors for different physical quantities (in //)
- Challenges:
  - Calculations are relatively expensive for higher resolutions
  - Bathymetry: strong sensitivity in shallow areas
  - Atmosphere (non-normality increases when introducing atmospheric coupling)